VIRIAL EXPANSION OF THE NUCLEAR EQUATION OF STATE*

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1. Introduction

In recent years the availability of new heavy-ion accelerators capable to accelerate ions from few MeV/nucleon to GeV/nucleon has fueled a new field of research loosely referred to as Nuclear Fragmentation. The characteristics of these fragments depend on the beam energy and the target-projectile combinations which can be externally controlled to some extent. This kind of experiments provides information about the nuclear matter.

This is very useful to make the best equation of state of nuclear matter. The conventional EOS provide only limited information about the nuclear matter : the static thermal equilibrium properties. In heavy ion collisions nonequilibrium processes are very important.

But *what is the best*
*Nuclear Equation of State***?**
(NEOS)

In this work new equations of state are proposed based on the conditions given by the critical phenomena of matter close to the critical temperature and density of a second – order phase transition.

What are the conditions?

- **Nuclear fragments**

After an energetic nucleus-nucleus collisions many light nuclear fragments, a few heavy fragments and a few mesons (mainly pions) .Thus the initial kinetic energy of the projectile leads to the destruction of the ground state nuclear matter and converts it into dilute gas ($\rho << \rho_0$) (these frozen-out fragments and their momentum distributions can be measured by detectors).

- **Phase transition**

Phase transition have been predicted theoretically through the study of the equation of state of nuclear matter. It is important to understand if such calculations are valid also in the case of finite nuclei.

Phase Transitions and Critical Phenomena

- If the temperature and density of our system falls into the unstable region, or even close to this region, it may split up into two phases.

  Theoretically this is also a consequence of the stability requirements if we allow for two coexisting liquid (L) and gas (G) phases we have one more free parameter in our thermodynamical problem, the volume fraction of the phases i=L,G. Now the requirement of the energy minimum leads to Gibb's criteria of phase equilibrium $P_L = P_{G,}$, $T_L = T_G$ and $\mu_L = \mu_G$.

$$\left(\frac{\partial \mu}{\partial T}\right)_P = -S \qquad \left(\frac{\partial \mu}{\partial P}\right)_T = V$$

  If the derivatives are continuous and the discontinuity is verified at higher orders, we will speak of second-order or continuous phase transitions

- In heavy ion reaction in principle we might reach the phase mixture region with arbitrarily high energy collisions in the subsequent quasi-adiabatic expansion [1] if the break-up density is sufficient low.

[1] L.P Csernai and H.W. Barz *Z. Phys.,* **A296,** 173 (1980).

• Critical Phenomena

We denote as critical phenomena the behavior of matter close to the critical temperature of a second-order phase transition. Continuous phase transitions are usually related to jumps in the symmetry of given system.

Examples of Second-order Phase Transition

- ✓Ferromagnetic-paramagnetic
- ✓Gas-liquid
- ✓Superconductivity
- ✓Superfluidity
- ✓QGP

Our problem ➜ QGP

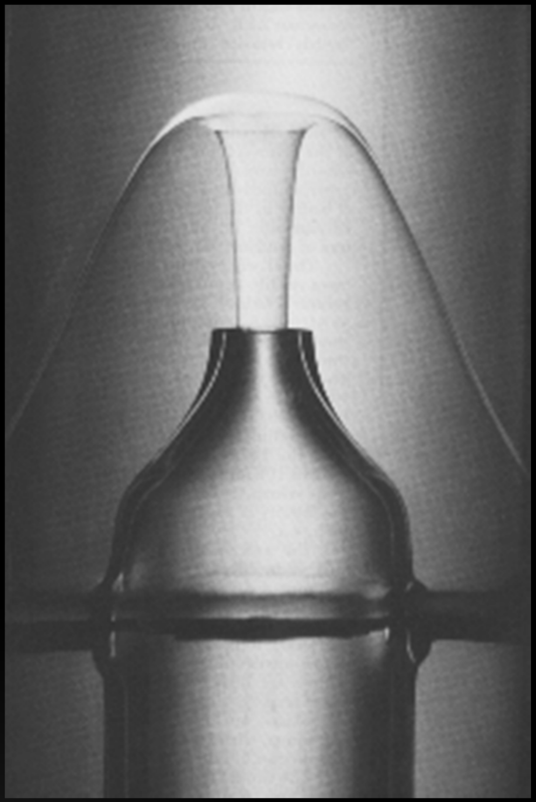


Fig. 1 Superfluid Helium fountain photographed by Allen in the 1970's

The helium flows up a tube and shoots In the air on being exposed a small heat source.

How we can study our problem?

ANS: Statistics Physics

- The properties of the system are different above or below the critical temperature and this fact is represented by the order parameter (OP)

▪Near the critical point some interesting themodynamic quantities such as the isothermal compressibility,

$$\chi_T = \left(\frac{\partial p}{\partial V}\right)_T$$

▪The specific heat c_p, etc. can be parametrized as power laws. The exponents of these power laws are the so-called "critical exponents"

$$(\rho_v - \rho_L) \sim |P|^{\frac{1}{\delta}}$$

Equation of State

▪And so on …

There are 6 critical exponents and there are some relations among them like Fisher, Rushbooke, Widom, Josephson derived from the scaling invariance of the free energy.

2.Theoretical Nuclear Overview

Conventional EOS

For a system interacting through two body forces having a short-range repulsion and a longer-range attraction the equation of state (EOS) resembles a Van Der Waals one. This is indeed the case for nuclear matter [2]. A popular approach is to postulate an equation of state which satisfies known properties of nuclei. The equation for energy per particle

$$E = 22.5\tilde{\rho}^{\frac{2}{3}} + \frac{A}{2}\tilde{\rho} + \frac{B}{\sigma+1}\tilde{\rho}^{\sigma} \qquad \tilde{\rho} = \rho / \rho_0 \qquad \rho_0$$

Kinetic energy of a free energy of a free Fermi gas

Potential interactions and correlations.

Normal nuclear density

[2] G. Sauer, H. Chandra and Moselu, *Nucl Phys. A,* **264**

- Let us consider for simplicity a classical system with an EOS in the form [3].

$$P = \rho^2 \frac{\partial E}{\partial \rho} + \rho T$$

The basic parameter is the (isothermal) compressibility which is defined as

$$K = 9 \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_0, T=0}$$

It is important to emphasize that the nuclear EOS strongly influences the phase transition and the phase diagram[1]

[1]The compressional energy is particularly important. When it is neglected [4] the resulting phase diagram may lead to pathological behavior, the matter at ρ0 and T=0 being in mixed phase.

[3]A. Bonasera et al., Rivista del Nuovo Cimento, **23,** (2000).
[4]T. S. Olson and W. A. Hiscock, *Phys Rev.* **C39**, 1818 (1989)

3.Nuclear Approach I

The three parameters *A*, *B* and σ are determinate using the conditions at $\rho=\rho_0$ we has a minimum, the binding energy is *E*=-15MeV and finally the compressibility is of order of 200MeV, as inferred from the vibrational frequency of the giant monopole resonance. Using these conditions, we get *A=356* MeV, and σ=7/6 .

Now if we modify this approach instead of compressibility the condition is given by the mean field potential

$$U(\rho) = A(\tilde{\rho}) + BA(\tilde{\rho})^{\sigma}$$

$$\frac{\partial U(\rho)}{\partial \rho} = 0 \quad \text{at} \quad \rho=\rho_0 \Rightarrow K = 225\text{MeV}$$

So at $\rho=\rho_0$ we have:

$a)\quad E = -15\text{MeV}$

$b)\quad \text{P} = 0$

$c)\quad K = 225\text{MeV}$

we get *A=-210* MeV, B=157.5 MeV and σ=4/3.

- For a nuclear system we expect to see a liquid-gas phase transition at a temperature of the order of 10MeV and at low density. Under these conditions we can assume that nuclear matter behaves like a classical ideal gas however this is just our ansatz.

- To calculate the critical point, we will impose the conditions that the first and second derivative of the conventional equation respect to ρ are equal to zero therefore we can obtain the critical temperature

- T_c =9MeV and density ρ_c=0.35ρ_0.

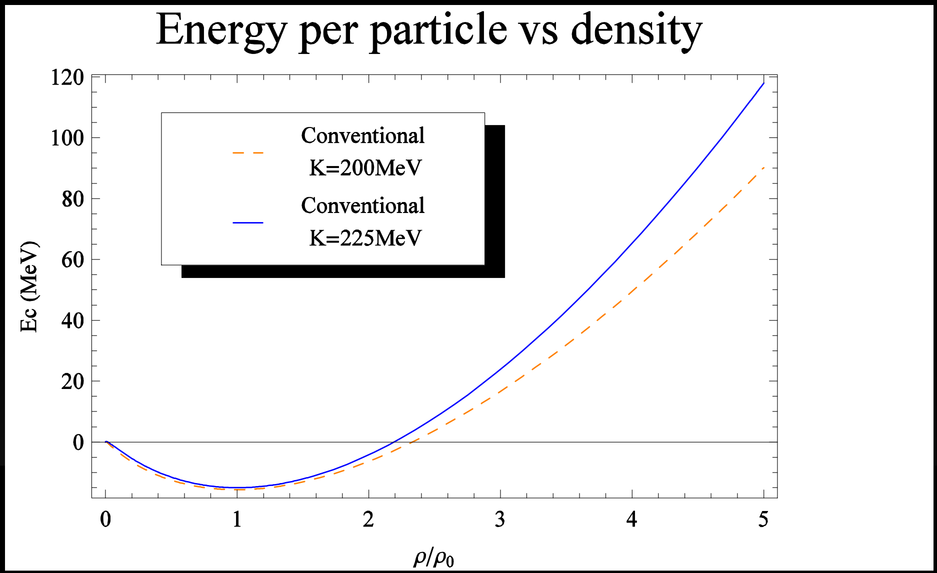


Fig. 2.0 Comparison between two conventional EOS

4.Nuclear Approach II

Virial EOS

In the virial expansion we are taking into account the interaction between pair of particles, and the subsequent terms must involve the interactions between groups of three, four, etc., particles. Now we will propose a new equation for the energy per particle.

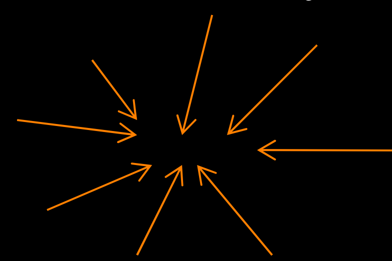
$$E = 22.5\tilde{\rho}^{\frac{2}{3}} + \sum_{n=1}^{k} \frac{A_n}{n+1}\tilde{\rho}^{n}$$

Kinetic energy of a free energy of a free Fermi gas

Potential interactions and correlations.

N-body interactions

- Let's consider three body forces *O*(ρ^3), take the same conditions of the conventional approach.
- Unfortunately, the solution has no physical sense because the energy diverges to minus infinity when densities approaches infinity.
- For the fourth order of our expansion *O*(ρ^4)



$$E = 22.5\tilde{\rho}^{\frac{2}{3}} + \frac{A}{2}\tilde{\rho} + \frac{B}{3}\tilde{\rho}^2 + \frac{C}{4}\tilde{\rho}^3 + \frac{D}{5}\tilde{\rho}^4$$

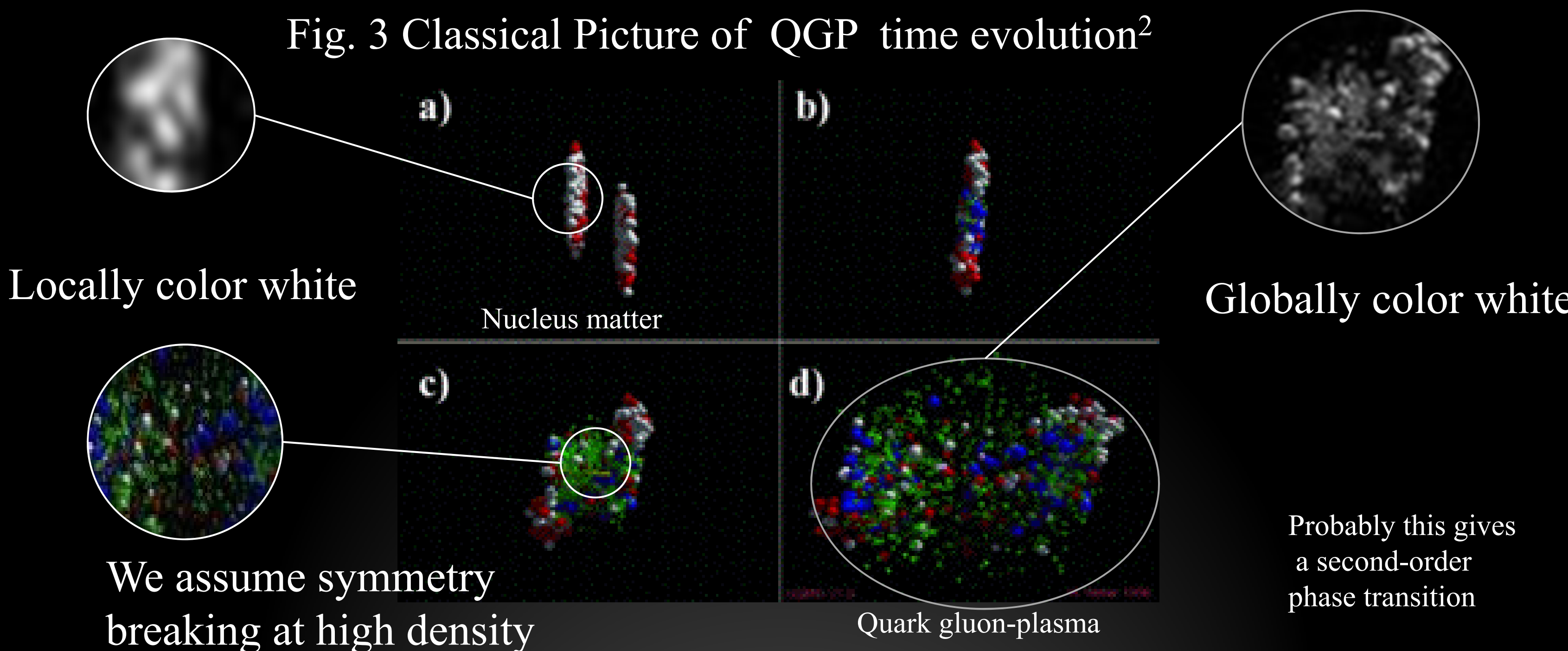
That would mean that the core would collapse and that is not possible

Don't forget that $\tilde{\rho} = \rho / \rho_0$

During the expansion and cooling of the universe the basic buildging blocks of our matter were created by combining quarks and gluons. Today the temperature and density of this phase transition can be computed quite precisely in extensive QCD lattice calculations. In addition there exists an experimental program at CERN (ALICE) or at RHIC. (PHENIX, STAR, Bhrams, Phobos experiments)

But whats going on in ultrarelativisic heavy ion collisions ?

Fig. 3 Classical Picture of QGP time evolution[2]



[2]Simulation of the time evolution of a collision between two Lorentz contracted heavy nuclei, *(ref. qgp.uni-muenster.de)*

- In accordance to the conditions used given by the Conventional EOS we can add two extra constrains based on the conditions given by the phenomena of matter close to the critical density of a second-order phase transition at zero temperature *T=0*.

- Then we will get five constrains:

$$a)\quad E(\rho_0) = -15\text{MeV}$$

$$b)\quad \text{P}(\rho_0) = 0$$

$$c)\quad K(\rho_0) = 225\text{MeV}$$

$$d)\ \left.\frac{\partial P}{\partial \rho}\right|_{\rho=\rho_c} = 0$$

$$e)\ \left.\frac{\partial^2 P}{\partial \rho^2}\right|_{\rho=\rho_c} = 0$$

Solving these nonlinear equations we get *A* , *B*, *C* and *D* and therefore we obtain the critical point ρ_c $=2.9354\,\rho_0$ at T=0

- For interactions between five particles $O(\rho^5)$ we obtain the same behavior like $O(\rho^3)$.

- If we take more variables on our expansion, until the sixth order and solve again this nonlinear system of seven unknown variables we get *A, B, C, D, E, F* . Then the critical point $\rho_c = 5.523\ \rho_0$ at T=0

SO if we want to add more Physics to get better results

- We can add the dependence of our EOS with the temperature . Taking into account our system like as a Fermi gas [4].

The temperature of the system can be derived experimentally [5] from the momentum fluctuations or particles in center of mass frame of the fragmenting source.

$$\frac{\rho}{\rho_0} = \left(\frac{a_0 T^2}{E^*} \right)^{\frac{3}{2}}$$

where

$$a_0 = A' / 13.3 \mathrm{MeV}^{-1}$$

[5] Wuenschel S, et al., *Nucl. Phys. A,* **843** (2b) 1-13(2010)

- The equation for energy per particle takes the form

$$E = 22.5\tilde{\rho}^{\frac{2}{3}} + \sum_{n=1}^{k} \frac{A_n}{n+1}\tilde{\rho}^n + a_0\tilde{\rho}^{-\frac{2}{3}}T^2$$

- The saturation point corresponds to the equilibrium point (at zero temperature) of nuclear matter hence characterized by vanishing pressure then

$$P = \rho_0\left[15\tilde{\rho}^{\frac{5}{3}} + \sum_{n=1}^{k} \frac{nA_n\tilde{\rho}^{(n+1)}}{(n+1)} + \frac{2}{3}a_0\tilde{\rho}^{\frac{1}{3}}T^2\right]$$

Then it is possible study the ⇨ Speed of sound

⇩

Compressibility

We connect the compressibility of this system with oscillations.

The compressional energy is particularly important because K gives a softer EOS and increase the critical baryon densities in cold matter.

The requirement of causality provides several theoretical constraints on the EOS [6], at high densities and limits the choice of the functional form of the compressional energy that can be used in phenomenological EOS

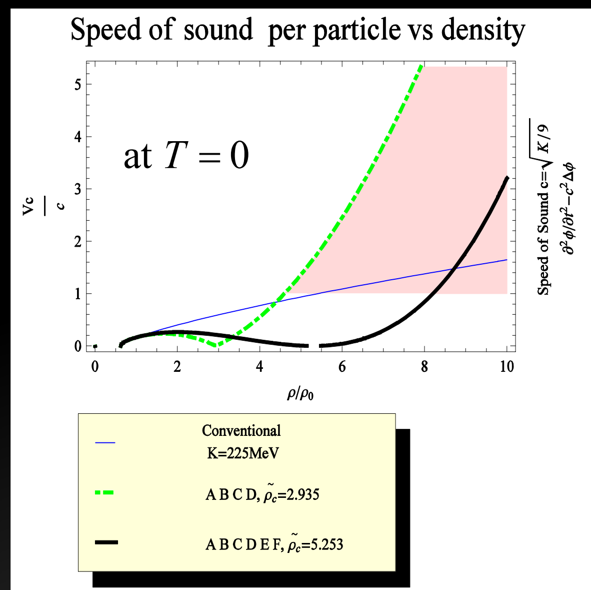


Fig. 4 The painted region breaks the principle of causality.

$$\partial^2\phi / \partial t^2 - c^2 \Box \phi = 0,$$

Where it is convenient to introduce the velocity potential v=gradϕ and this potential must satisfy

$$c = \sqrt{\partial P / \partial \rho}$$

[6]T. S. Olson and W. A. Hiscock, *Phys Rev.* **C39**, 1818 (1989)

Compressibility per particle vs density

Speed of sound per particle vs density

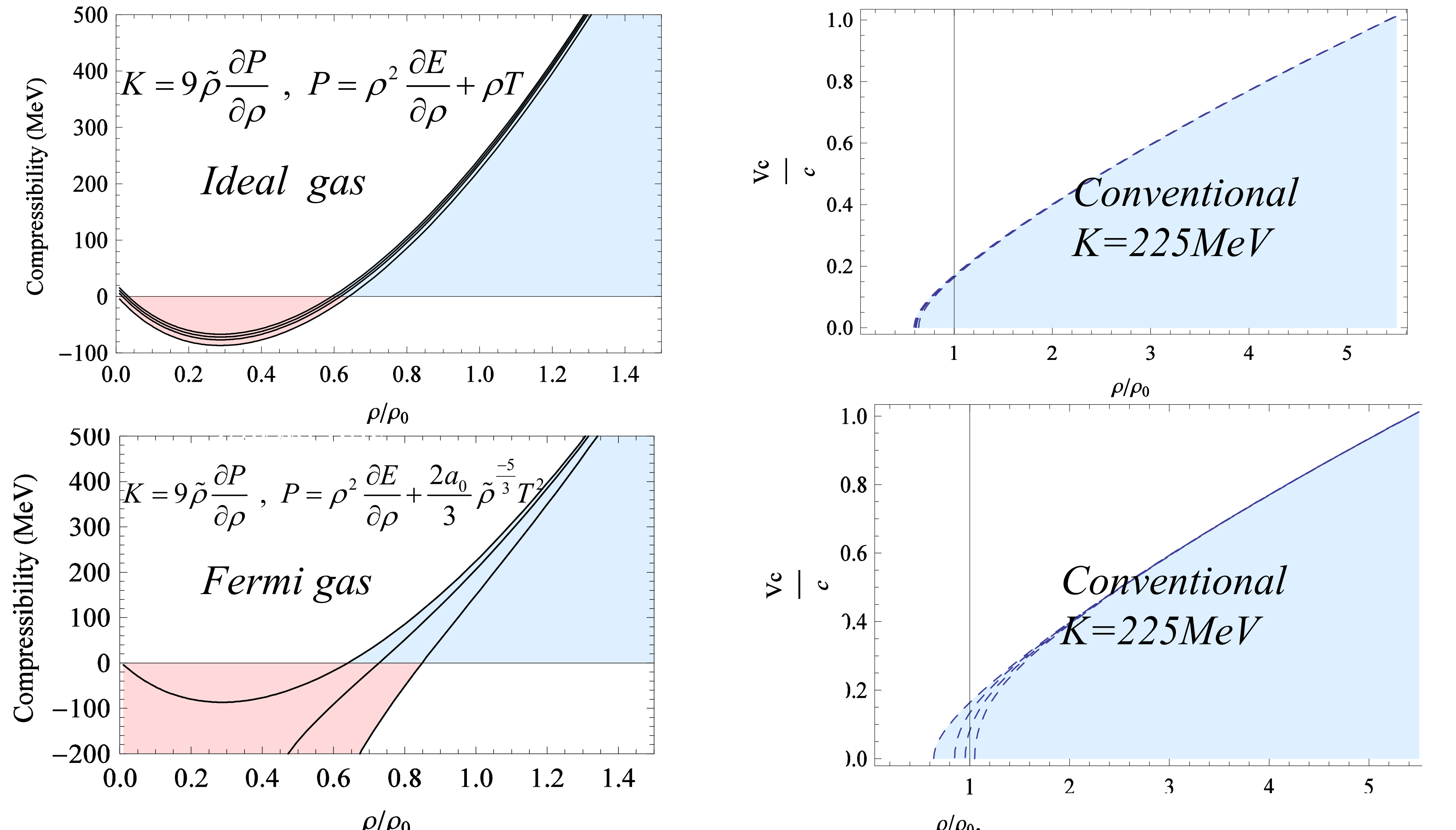


Fig. 5. Some properties or our EOS at some temperature (0-15MeV). The first four plots are the comparison of the compressibility and speed of sound between classical ideal gas and Fermi gas for the conventional EOS.

Compressibility per particle vs density

Speed of sound per particle vs density

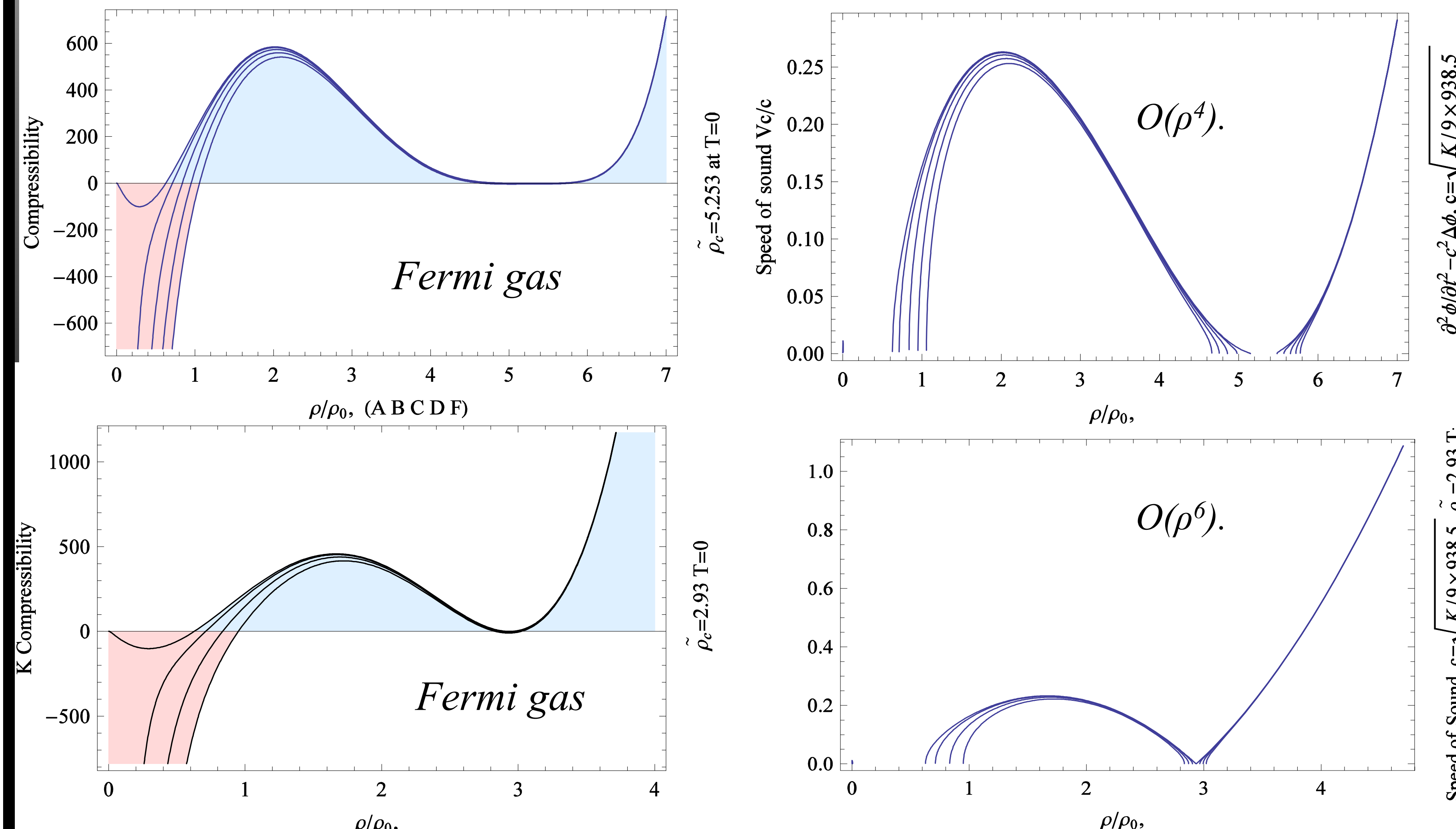


Fig. 6 Some properties or our Virial EOS at many temperatures near to T=0 (0-15MeV)

4. Theoretical Results

- For the case of fourth order $O(\rho^4)$.of our EOS, the critical temperature and density have the values

$$T_c=18.05\text{MeV and } \rho_c=0.3724\,\rho_0 \quad \text{respectively.}$$

This could be the critical point for the second-order phase transition In this approach we are assuming interactions between pair of particles and between groups of three and four particles. This could be fully justified by the fact that nucleus are made of quarks and gluons.

- For higher terms, for the sixth order $O(\rho^6)$.the critical temperature and density are

$$T_c=17.932\text{MeV and } \rho_c=0.3717\,\rho_0.$$

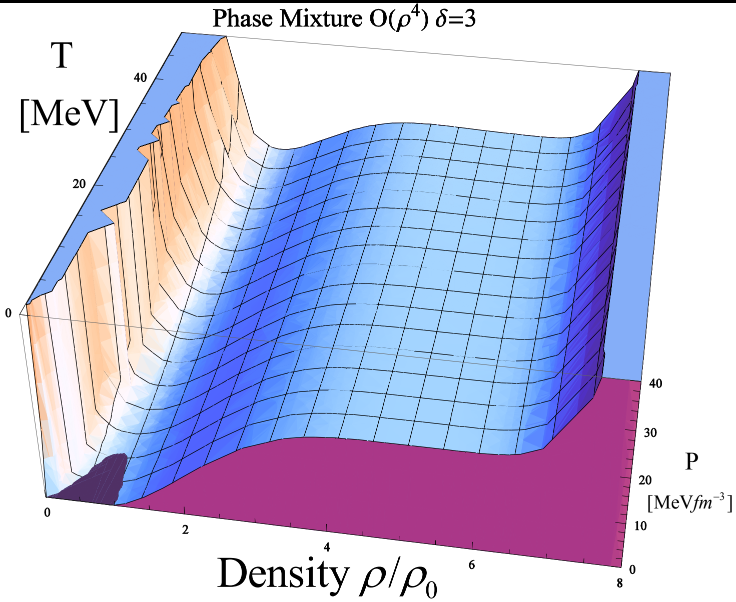


Fig. 7.0 Equation of state surface for a nuclear system with a second-order phase transition. The pressure is reduced by phase transition, when the density and temperature reach the mixed phase region. The pressure increases again only when the pure quark gluon phase is reached.0

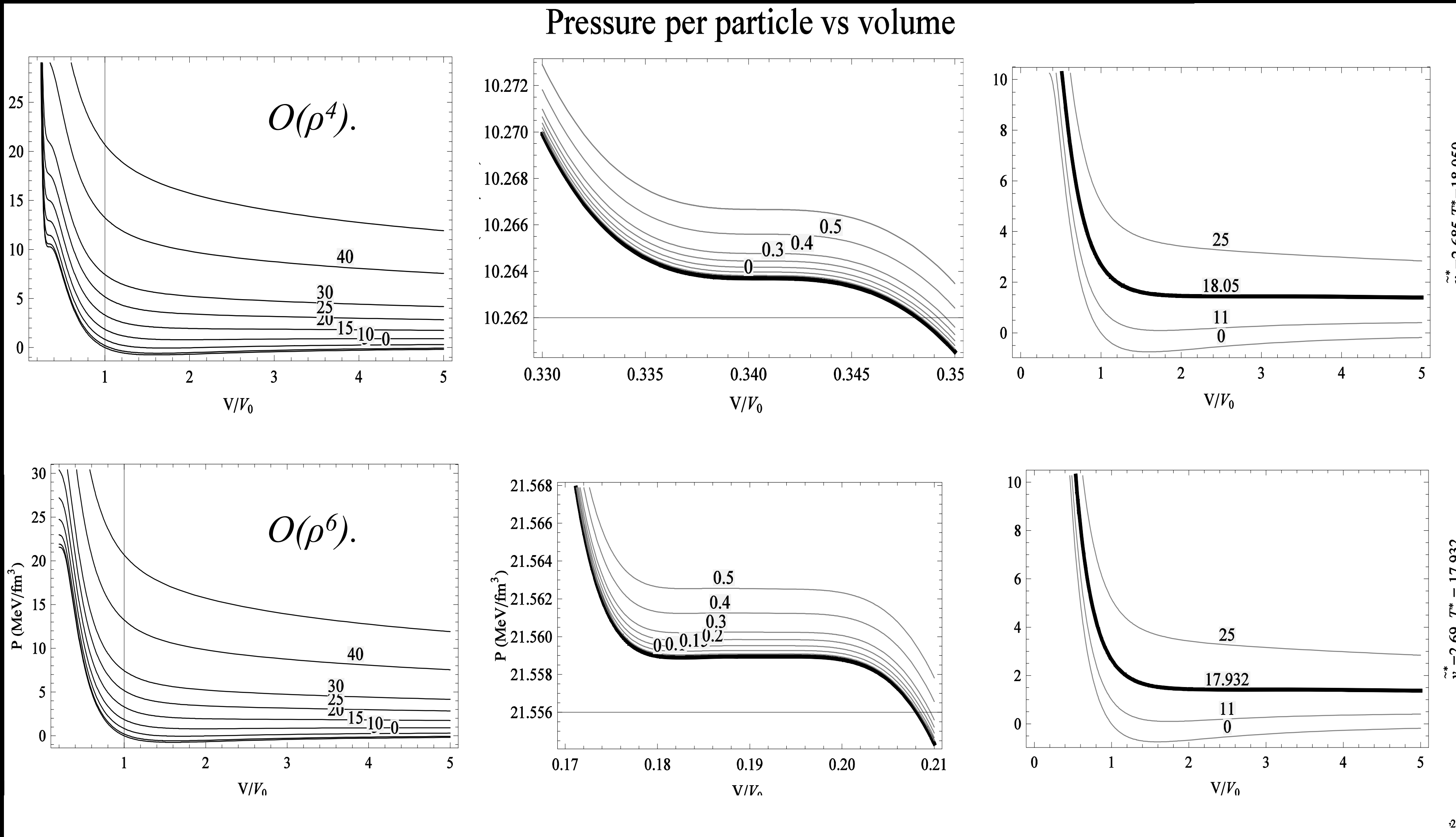
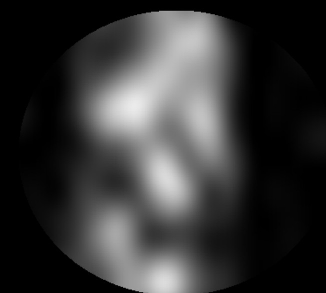


Fig. 8 Behavior near to the critical point. The pressure per particle of nuclear matter as a function of volume at some temperatures

- If we take the free energy in terms of a power series and we consider an external field P_c and we stop our expansion at the fourth order *O(ρ⁴)* , then introduce the parameter $\eta=(V - V_c)$ it is possible find the minimum at the critical temperature .

$$\eta_c \approx \left(\frac{6P(V_c)}{\partial^4 E \big/ \partial V^4 \big|_{Vc}} \right)^{\frac{1}{3}}$$



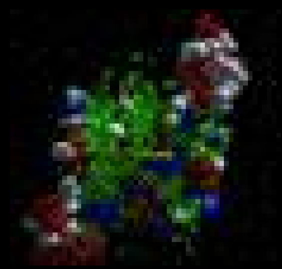
Then $O(\rho^4)$ has the critical exponent

$\delta=3$

Now if we take more terms stopping our expansion at the sixth order $O(\rho^6)$, and we assume that the second until sixth derivatives of the energy with respect to volume are zero we obain

that $O(\rho^6)$ has the critical exponent

$\delta=5$



5.Conclusions

The virial expansion of the nuclear equation of state reproduce some elementary properties of our nuclear system.

✓In this work we have determined the critical density for the second order phase transition at T=18 MeV. ρ_c =0.37 ρ_0

For $O(\rho^4)$. we calculated δ=3 and $O(\rho^6)$ δ=5 . Experimentally it is known that the value of the delta must be in the range δ ~4-5; that means that for $O(\rho^6)$ are in accordance with experimentally acceptable values.

✓. We found that for odd orders in this approach are not suitable to describe the basic properties of our core system give infinitely negative energies.

✓For even orders , we have more acceptable behavior of the energy per particle as function of density.

✓It could be possible that the collective character in regions with high density of particles are associated with even interactions[7]

[7]A. Bohr and B.R. Mottelson, " Collective and individual-particle aspects of nuclear structure", Mat. Phys. Dan. Vid. Selks. **27** No 16 (1953).

For the future

- It would be of interest to see how our equation of state affects the microscopic dynamical evolution of the nuclear system.
- Make computer simulation of collisions using the proposed EOS
- Conduct further studies of nuclear matter at densities $\rho_c \sim 0.35\ \rho_0$ - $0.37\ \rho_0$ with temperatures T ~ 9 -18 MeV to test the predictions of the EOS.
- Check the virial approach with the relativistic mean field at higher density.
- Verify the parity (odd or even interactions) dependence in higher density nuclear systems.
- Verify correlations between symmetry breaking of colors and transfer phase transitions.
- Work and work…

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